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SELECTED PROBLEMS OF DETERMINING THE EFFECTIVENESS OF THE SEPARATION TECHNOLOGICAL PROCESSES OF MINERAL ENGINEERING

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Evaluating the rate of reaching the assumed aim of any activity is the ground of verification of correctness of conduct assumed for its implementation. This evaluation is also the basis of verification of the algorithm of this conduct, including its modifying in order to reach the optimum state of the assumed goal. The procedures used for such an evaluation used to be described as a study of effectiveness of activities in question and its result directly as effectiveness. In case of technological processes, including the basic group of operations of mineral engineering, effectiveness is usually determined as a numerically expressed relation of really obtained process results to the results assumed, forecast or theoretically possible to be obtained. The variety of formulations of detailed assumptions of processes occurring in this discipline formulate the need of significant differentiation not only of the methods of evaluating its effectiveness but also precise determination of the very notion of effectiveness in the concrete conditions of implementation of the technological process. The work contains a discussion of this problem.

Key words: technological process, process aim, process effectiveness

BASIC CONDITIONS OF TECHNOLOGICAL PROCESSES, THEIR AIDS AND EVALUATION

Heterogeneity of tasks performed by means of operations and processes (sets of operations) of mineral engineering requires the application of not only a very well developed range of technological procedures based upon the use of numerous properties of the material subjected to processing but also a differentiated approach to
determining the aims of this processing as well as the evaluation of the obtained results.

The subject matter of these remarks is constituted by separation processes, which result in obtaining at least two products of mutually differentiated properties from the feed. This group comprises the majority of processes determining the economic application of almost all mineral raw materials, i.e. primary and secondary.

The above description of separation processes is sufficient for the most general approach yet it is highly imprecise in relation to detailed requirements, which are formulated as implementation aims\(^2\). It should be remarked here that all rational considerations concerning mineral engineering must take into account its utilitarian character and the mentioned various solutions, necessary to achieve the assumed aims. The latter ones, unit by unit, can be formulated as:

- obtaining products of assumed properties, most often the contents of certain components (elements, grain classes, other phases differentiated in a certain way) which are concentrated (selectively or collectively) in concentrates (components differentiated due to the chemical composition – metals, combustible substances and others), grain size fractions (grain classes), and also the minimizing of these contents in a given product not apt for further processing or, generally – at least now – useless (secondary materials, waste),
- maximization of recovery of these components by means of introducing them into appropriate concentrates – increase of the utilization rate of the raw material,
- eliminating a certain component (components) from the concentrate of another component – obtaining the required effectiveness of separation of concentrates,
- obtaining a possibly large number of useful components of the multi-component raw material – complex (full) utilization of the raw material (Sztaba 1970),
- maximization of economic effects of the raw material utilization – obtaining the highest profits while providing the assumed product properties, and also many other variants of the assumed aims, in particular different listings of unit aims, given as examples. The need of constructing such listings occurs first of all in the cases of multi-product processing of raw materials. This concerns mainly the multi-component raw materials, for example the polymetallic metal ores, but also the complex utilization of any other raw materials and producing from the material with one separated component a few concentrates of different uses and thus differentiated properties, for instance a rich metal concentrate for pyrometallurgical processing and a poorer one for hydrometallurgy. It should be observed that when more than one “useful” product is obtained, their yields, costs of obtaining and also commercial values are generally different which requires taking into account the process results, especially when evaluating the economic effects.

The outlined conditionings of mineral engineering processing significantly affect the possibilities of evaluating their results and the choice of the method of such an

\(^2\) formal descriptions of separation and non-separation processes in (Sztaba 2002b)
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evaluation. It should be stressed that a limited range of information is left to be the starting point of such an evaluation. It usually comprises the qualitative characteristics and content of separated components in the feed and at least in the selected products, sometimes also in their separate fractions (e.g. grain classes); rarely direct information about mass expenditures (yields) of products, unit costs of process execution and commercial values of products. Gaining additional information, though more and more possible, results in additional costs, not always to be confirmed by reaching the increased value of production3.

PRINCIPLES OF DETERMINING THE PROCESS EFFECTIVENESS

As it has been already mentioned, every technological process is performed with the assumed aim and determining the effectiveness of this process is used to evaluate the degree of this aim. Determining the effectiveness of processes and methods of its evaluation have been studied theoretically and practically by mineral engineering (mineral processing).

The notion as such was named differently (Drzymała 2002, previously many others), many methods of calculating the effectiveness were proposed (often adapted to single cases) (Barskij, Plaksin 1967, Barskij, Rubinštejn 1970, Stepiński 1964 and others). The terminology standard introduced in Poland (Polska Norma 1999) recommends univocally the use of the notion of effectiveness.

In case of the principal group of mineral engineering processes effectiveness was assumed to be determined as a numerically expressed relation of really obtained process results to the assumed, forecast or theoretically possible results.

The general definition of effectiveness can be presented as (Sztaba 2002b)4:

\[ S = E = \frac{W_r}{W_0} \]  

where:

- \( W_r \) – obtained result,
- \( W_0 \) – expected or theoretically possible result.

Equation (1) can be treated as a general definition of effectiveness. In technological applications only these cases are considered in which the values \( W_r \) and \( W_0 \) assume numerical values. Practically, most often the value of effectiveness, calculated in such a way, is assumed to be a percent evaluation of success in aiming at reaching the value of \( W_0 \), multiplying the fraction in expression (1) by 100.

3 the problem of evaluation of the economic value of information has not been practically solved despite a general statement, given here; it is of a very broad range, concerns not only the discussed processes and is not the subject-matter of the present considerations,

4 in the definition formulas (1), (2.1), (2.2) and in descriptions the author applied a more general designation of effectiveness – \( S \), beside the more popular one – \( E \) and, next, he uses \( E \) all the time.
THE CHOICE OF THE BASE OF REFERENCE AND THE SUBJECT OF EFFECTIVENESS CALCULATION

The assumption of the value \( W_0 \) is of principal significance for the result of calculations of the numerical value of effectiveness (further on, shortly, effectiveness). The majority of the applied and proposed methods of effectiveness calculating assume openly or more often assumingly that the aim of the process is to obtain that an ideal result; in case of the discussed separation processes, the ideal accurate separation of the selected feed components, i.e., among others, the maximum content of these components in their corresponding products. All known formulations of the theory of separation indicate agreeably that reaching such a result would occur at a very high outlay of energy (practically limitless high). In all separation processes the increase of separation accuracy is obtained at the cost of the progressively growing expenditure of energy, i.e. also the costs. Therefore, in real circumstances, such requirements are never applied, even at the separation of products of high quality standards, such as abrasive micropowders. Thus it should be assumed that the value \( W_0 \) should be, except for totally exceptional cases, the expected result of the process. This variant is generally taken into consideration in the applied methods of calculating the effectiveness by means of introducing the tolerance ranges for the obtained results of its evaluation.

Determining the subject-matter of effectiveness evaluation creates another problem. To present this, it is possible to use the simplest case of effectiveness evaluation according to the content – \( a_1 \) – of the selected component in the appropriate product. Then \( S = a_1 \). Attributing this expression the features of effectiveness equals an assumption that it represents the results of applying formula (1) and thus that, in fact, it is the expression \( S = a_1 / 1 \) and, consequently, \( W_0 = 1 \). It can be accepted, for instance, in case of the process of grain classification where such grains are separated which can belong only to one of a few mutually separable classes, or, for example, in case of coal enrichment, if we assume the occurrence of grains of pure mineral substance and \( a_1 \) is its content. On the other hand, such an assumption for the evaluation of effectiveness of producing the metal concentrate would mean that obtaining pure metal in the enrichment process could be assumed. This can be attributed only to the entire processing process whose evaluation is not grounded according to the results of enrichment exclusively. Such an assumption would be grounded in this case:

\[
S = E = \frac{a_1}{a_{\text{max}}},
\]

(2.1)

where \( a_{\text{max}} \) – metal content in the mineral being its carrier.

Taking into account all simplifications and conventionalities of this example it can be stated according to it that the phase significantly subjected to separation, i.e. the grain class, metal-bearing mineral, “pure” grains of crude coal, should be a real
subject of separation effectiveness evaluation. It contains the minimum impurities of mineral substance (and the grains and waste rock – of combustible substance). On the other hand, however, introducing the expected, e.g. required by the buyer, value of \( a_{\text{prod}} \) into the denominator of the expression for \( S \) results in the simplest and practically applied principle of agreement between the real concentrate quality with the assumed one.

\[
S = E = \frac{a_1}{a_{\text{prod}}},
\]

from which the lack of purpose of concentrate production appears \( a_1 > a_{\text{prod}} \) then \( S > 1 \) with the unnecessary outlay of energy.

**ASSUMPTIONS OF SYSTEMATIZATION OF EFFECTIVENESS EVALUATIONS**

The formerly stressed various requirements concerning both the shaping of the process results and rules of evaluation resulted in the origin of very many methods and means of such an evaluation (Barskij, Plaksin 1967; Barskij, Rubinštejn 1970; Stępiński 1961, 1964; Sztaba 1983-2001, 1998a, 2000a, 2000b, 2000c, 2002a and others). A mainly practical significance of these methods is the cause of a few attempts of a purely formal approach to its forming (Drzymała 2002), separated from a very differentiated demand.

Many authors, including the above ones, pointed out the possibility of differentiating a few basic groups of the discussed methods, assuming as a selecting criterion the variant interpretation of the basic notion of effectiveness, generating the origin and development of the methods of approach with the application, of course, of the formerly discussed range of information about the results. The basic evaluation groups (evaluation criteria) were differentiated:

- **principal** (very vast literature, despite the previously quoted: Sztaba 1956a, 1956b, 1983, 1993a, 1993b, 2001; Tumidajski 1993 and many others)
  - technological,
  - statistical,
  - economic,
- **but also**
  - power engineering (Sztaba, Tora 1987; Tora, Sztaba 1983) and
  - thermodynamic (Barskij, Plaksin 1967; Barskij, Rubinštejn 1970),
- **stressing the approaches** (Barskij, Plaksin 1967; Barskij, Rubinštejn 1970 and others):
  - static and
  - kinetic.

\(^{5}\) the information about the process results does not exhaust the description of its course conditions, they both constitute jointly a basic for the construction of process models which, among others, formulate the foundation to create specific effectiveness evaluations and which are not the subject-matter of this paper.
The basic characteristics of the mentioned groups are widely presented in the quoted works, which grounds its neglecting in the present one. Following, certain additional features of the selected effectiveness evaluations will be indicated and discussed, mainly technological, which are most often applied both in industry and science.

SELECTED REMARKS ON THE PROCESS EFFECTIVENESS EVALUATION METHODS

TECHNOLOGICAL EVALUATIONS FOR THE ENTIRE PROCESS MATERIAL

The requirements of industry contributed to the most intensive development of effectiveness evaluation methods in the “technological” group, i.e. using the values directly corresponding to the methods of presenting technological characteristics of the feed and process products. They are used both for the evaluation of quality of products and the rate of utilization of feed components. They also constitute the base of evaluation of operations of mineral processing plants. The heterogeneity of detailed aspects of performing such evaluations resulted in their specific specialization, i.e. a possibility of differentiating three distinct subgroups of evaluations:

a) qualitative,
b) quantitative,
c) general.

The results of the performed process, determined for the entire processed material, are most often the subject of evaluation, as it is indicated by the title of this subchapter. Yet, in case of a more precise process study, and, especially, when statistical descriptions and evaluations are introduced, it is necessary to trace the behaviour of the feed grains during the process, differentiated according to certain qualitative criteria (grain size, rarely shape, density and possibly other distinctive features). Subchapter 3.2 contains general remarks concerning methods of conduct in such cases.

The elementary separation process of the feed with one distinctive component was assumed to be an example for discussing the characteristics of selected evaluations. The component content in the feed was $a_0$. There were two products, 1 (e.g. “concentrate” of $a_0$ content) and 2. (e.g. waste of $a_0$ content) in the feed $a_1 > a_0 > a_2$. The yields ($\gamma$) of products are calculated in the well-known way according to the component balance (Stępiński 1964):

$$\gamma_1 = \frac{a_0 - a_2}{a_1 - a_2} \quad (3.1)$$

$$\gamma_2 = 1 - \gamma_1. \quad (3.2)$$
All values are given in fractions. Practically, percent values are used, the method of mutual transformations is obvious. In the formulas used as examples the most popular denotation $E \equiv S$ is used, adding differentiating numbers.

**Note a)** This is the most numerous subgroup (conventional denotation $E'$), using mainly the qualitative features of products. They comprise the following evaluation methods:

- only quality of products, out of which each one is evaluated separately, here the examples are simple evaluation methods, described in chapter 2.2 with formulas (2.1) and (2.2),
- methods of separation selectivity – applied for the differentiation rate of products quality – their construction is based on the non-negative difference of content, e.g.:
  \[ E'_{5} = a_{1} - a_{2}, \]  
  or  
  \[ E'_{7} = \frac{a_{1} - a_{2}}{a_{0}} \] (“Truszlewicz’s index”),

or their quotient (>1), for instance:
  \[ E'_{6} = a_{1} / a_{2}, \]  
  or  
  \[ E'_{8} = \frac{a_{1} \cdot (1 - a_{2})}{a_{2} \cdot (1 - a_{1})} \] (“Gaudin’s index”)

- methods of rate of approximation to the largest possible differentiation of content in the selected product and feed:

  for product 1.:  
  \[ E'_{9_{1}} = \frac{a_{1} - a_{0}}{a_{\text{max}} - a_{0}} \] (8.1)
  for product 2.:  
  \[ E'_{9_{2}} = \frac{a_{0} - a_{2}}{a_{0} - a_{\text{min}}} \] (8.2)

  where $a_{\text{min}}$ – the least possible content in product 2. (e.g. the so-called value of background).

  Formula (2.1) shows the formerly mentioned justification of including permissible deviations of value $a_{1}$ from $a_{\text{max}}$ (or $a_{\text{min}}$) in the effectiveness evaluation. If product 1. is allowed to control “impurity” caused by a separate material in the amount $\delta_{1}$ and, analogically, product 2. contains it in the amount $\delta_{2}$, this formula is transformed:
for product 1, in the form: 

$$E_{t_1}^{\delta} = \frac{a_1}{a_{\text{max}} - \delta_1} > E'_{t_1} = \frac{a_1}{a_{\text{max}}}$$, (9.1)

for product 2, in the form: 

$$E_{t_2}^{\delta} = \frac{(a_{\text{max}} - \delta_2) \cdot a_2}{a_{\text{max}}} < E'_{t_2} = \frac{a_2}{a_{\text{max}}}$$, (9.2)

Similarly, other formulas can also be transformed and values \(\delta_1\) and \(\delta_2\) can be taken into consideration.

**Note b)** Practically, the only evaluation in the quantitative group (conventional symbol \(E''\)) is the recovery, \(E''_1 = \varepsilon\), representing the utilization rate of the certain part of the component included in the feed. Therefore it is the most important indicator of evaluation of the raw material utilization rate, used to evaluate the quality of operating of the system of the processing plant. The well-known formula is used to calculate the recovery: 

$$\varepsilon_1 = E''_1 = \frac{a_1 \cdot (a_0 - a_2)}{a_0 \cdot (a_1 - a_2)} = \gamma_1 \cdot \frac{a_1}{a_0}.$$ (10)

The recovery calculation can be disturbed when there is a partial change of the material characteristics in the process course. A good example is constituted by a part of flow classification processes, more seldom by sieving, in which there are significant tangential forces between material grains and machine elements (e.g. hydrocyclones, sedimentation centrifuges, high-movement sieves, etc). In these cases the total number of fine classes in the sum of product is larger than in the feed at the cost of coarser classes. If the total increase of the content of the fine class (evaluated component) in relation to the feed is \(\Delta\), then its resulting content in the feed is \(a_0 + \Delta = a_0^*\) and the formula of recovery will be:

$$\varepsilon_1^* = E''_1^* = \frac{a_1 \cdot (a_0^* - a_2)}{a_0^* \cdot (a_1 - a_2)} = \gamma_1^* \cdot \frac{a_1}{a_0},$$ (10.1)

where \(\gamma_1^*\) – corrected value of yield of product 1.

The group of quantitative evaluations comprises also a more complex evaluation of separation accuracy of respective material components, separated to appropriate concentrates. Here the selection indexes are used which are calculated as geometric means of the relations of recovery and rejection (filling up recovery \((1 - \varepsilon)\) determine which part of the total amount of the given component contained in the feed is found outside the appropriate concentrate) of both considered components. If we assume that two components, A and B, are separated into appropriate concentrates and their recovery are marked in the component A concentrate as \(\varepsilon_{AA}\) and \(\varepsilon_{AB}\) respectively, then the selection of component A off component B is described by the expression:
There are still other methods of calculating the selection index, depending on the evaluation variant of the technological system (Stepiński 1964).

Note c) The method presented by Hancock in 1918 and usually connected with his name has the principal significance in the subgroup of general evaluations – symbol E. Regardless Hancock’s propositions, there are at least several independent works (Barskij, Rubinštejn 1970; Sztaba 1993b) whose authors, starting from seemingly different assumptions, obtained the same result in the form of the formula:

\[ E_i = \frac{(a_0 - a_2) \cdot (a_1 - a_0)}{a_0 \cdot (a_1 - a_2) \cdot (a_{\text{max}} - a_0)}, \]  

in which, especially in case of applying in the grain classification processes and when there are no precise data, \( a_{\text{max}} = 1 \) is often assumed. The relation between evaluation (12) with recovery (10) makes this evaluation susceptible to changes of material composition in the course of the separation process. Taking into account the same assumptions of this conditioning as in the case of recovery, it is obtained, analogically to formula (10.1):

\[ E_i^* = \frac{(a_0^* - a_2) \cdot (a_1 - a_0^*)}{a_0^* \cdot (a_1 - a_2) \cdot (a_{\text{max}} - a_0^*)}. \]

TECHNOLOGICAL EVALUATION FOR FRACTIONS OF THE PROCESSED MATERIAL

As it was mentioned in the introduction to subchapter 3.1, there is a need (in some research projects, in the application of some statistical evaluation methods) of studying the behaviour of separate fractions during the process and these are numbered successively 1, 2, ..., i, ..., n, which can be separated in the processed material. The mechanisms of such behaviours are in agreement with the behaviour of non-fraction products. Therefore for their evaluation the methods of process effectiveness evaluation are applied with the application of values used during the technological process evaluation as input ones. These values concern the fractions being separated. It is assumed that such evaluations are marked with small letters; \( e_i \) – for the i-th fraction, with other discriminants as for process evaluations. For instance, formulas (7), (10) and (12) take the forms:

\[ e_{ij} = \frac{a_{ij} \cdot (1 - a_{ij})}{a_{ij} - a_{ij}}. \]
\[ e_{i_1} = \varepsilon_i = \frac{a_{i_1} \cdot (a_0 - a_{2_1})}{a_{0_1} \cdot (a_{i_1} - a_{2_1})} = \gamma_i \cdot \frac{a_{i_1}}{a_{0_1}}, \]  
(10.2)

\[ e_{i_1} = \frac{(a_0 - a_{2_1}) \cdot (a_{i_1} - a_0)}{a_{0_1} \cdot (a_{i_1} - a_{2_1}) \cdot (a_{\text{max}} - a_0)}, \]  
(12.2)

where \( a_{s_i} \) – value \( a_s \) for the \( i \)-th fraction.

Special attention should be paid to the recovery of the \( i \)-th fraction in the selected product (10.2), identical to the number of separation – \( \varepsilon_i \equiv \tau_i \) – the basic value occurring in statistical descriptions and evaluations of results of separation processes, determining the possibility of transfer of grains of certain properties to the chosen product (among others: Sztaba 1956a, 1956b, 1983-2001, 1993b; Stepiński 1964; Tumidajski 1993).

**OTHER SELECTED REMARKS ON THE CONSTRUCTION OF EFFECTIVENESS EVALUATIONS**

Certain separation processes include limitations for a free separation of certain grain groups. “Difficult grains”, taken into consideration in the sieving process, are such an example. Their occurrence, significant for the process course, requires including the evaluation methods in construction (Sztaba 1993b and others).

At the beginning of the nineties of the previous century the notional identity of the results of separation processes with the phenomenon of natural segregation of grained materials was pointed out. The latter ones were heterogeneous due to at least one feature, which could be a separation feature\(^6\). It enables the application of the segregation rate achieved in the products of such processes for the evaluation of their effectiveness (Sztaba 1993a, 1998a, 2000a).

The significance of complex utilization of mineral raw materials, stressed in the introduction, being one of important conditions of reaching the sustained economic and social development, evoked the need of determining the principles and methods of evaluation of multi-product separation processes, most often the multi-component input raw materials, including the secondary ones. The present propositions assume the calculation of effectiveness of such processes according to partial evaluations, performed for selected material components, taking into account their participation in the feed and also weights considering their economic value, including the quality values, market unit values of respective concentrates as well as the costs of their production and possibilities and costs of managing the resulting secondary products or waste (Sztaba 1983, 2000b, 2000c, 2001, 2002a). The research to solve this group of

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\(^6\) the feature, most often physical, of grains whose differentiated values condition directing them to respective products of the process
tasks requires special attention; especially that one should foresee the necessity of considering additional evaluation elements in the form of environmental, social and other conditionings of the sustained development.

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i surowców wtórnych – użytkowanych praktycznie we wszystkich gałęziach produkcji przetwórczej z ich niezliczonymi wariantami założeń i celów szczegółowych, wywołuje potrzebę znacznego różnicowania nie tylko metod oceny ich skuteczności, lecz też modyfikowania interpretacji podstawowego pojęcia skuteczności w konkretnych warunkach założeń i przebiegu praktycznie każdego procesu technologicznego. Opracowanie zawiera przegląd podstawowych wariantów takich wymagań i przedstawia propozycje – w części już wykorzystywane – dostosowywania do nich sposobów szczegółowych określania skuteczności procesów. Przedstawia również wybrane, rzadziej zauważane okoliczności wpływające na ocenę skuteczności procesów, w tym dyskusję poziomu odniesienia takiej oceny, uwzględniania dopuszczalnych tolerancji jakości produktów, przypadków zmiany w trakcie procesu niektórych właściwości pierwotnych nadaw. Ograniczając rozważania do wybranych zagadnień tzw. ocen technologicznych, wskazuje się na ich związki z innymi podstawowymi grupami ocen: statystycznych i ekonomicznych, a także na kierunki rozwoju metod oceny niezbędnego dla sprostania zadaniom kompleksowego wykorzystania surowców, warunkującego między innymi realizację zasad zrównoważonego rozwoju gospodarki i społeczeństwa.